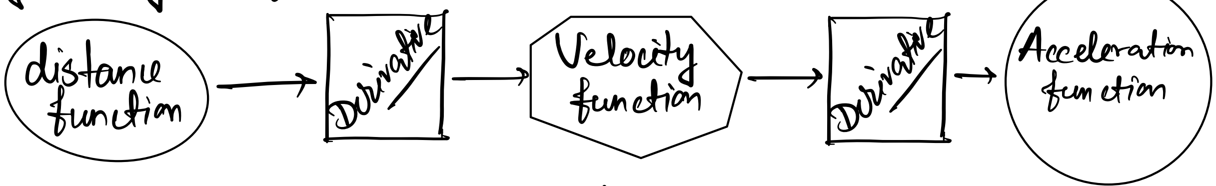
Higher Order Derivatives

Since derivative of a function is itself a function, then we might again find the derivative of a derivative.

Eg.


$d(t)$ $\qquad v(t) = d'(t)$ $\qquad a(t) = v'(t)$
$= d''(t)$.

In general: $f''(x),\ f'''(x),\ f^{(4)}(x),\ \ldots,\ f^{(n)}(x)$

$y''(x),\ y'''(x),\ y^{(4)}(x),\ \ldots,\ y^{(n)}(x)$

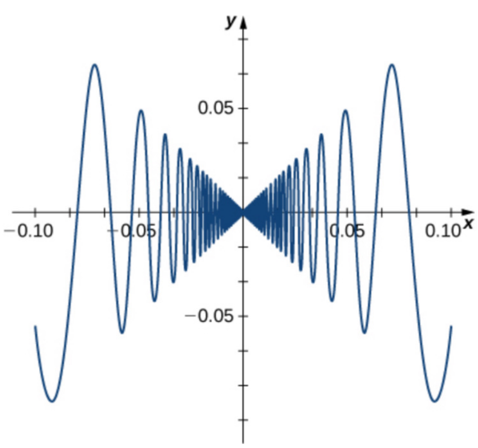
$\frac{d^2}{dx^2}(f(x)),\ \frac{d^3}{dx^3}(f(x)),\ \ldots\ldots,\ \frac{d^{(n)}}{dx^{(n)}}(f(x))$

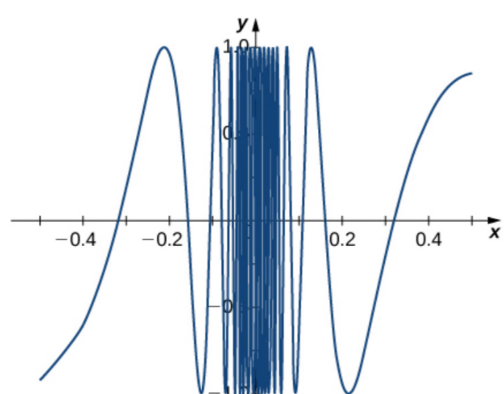
$\frac{d^2y}{dx^2},\ \frac{d^3y}{dx^3},\ \ldots\ \ldots\ldots,\ \frac{d^ny}{dx^n}$.

Note: $\lim\limits_{x \to 0} \sin\left(\frac{1}{x}\right)$ doesn't exist.

Ex. $f(x) = \begin{cases} x \sin\left(\frac{1}{x}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$

Graph of $\sin\left(\frac{1}{x}\right)$





Differentiation rule:-

① Constant Rule: If $f(x) = c$, some constant. Then

$$f'(x) = 0 \quad \text{or} \quad \frac{d}{dx}(c) = 0 \quad \text{or} \quad (c)' = 0$$

② Power Rule: If $f(x) = x^n$, $n \neq 0$ is an integer, then

$$f'(x) = n\,x^{n-1}.$$

Eg. $f(x) = x^{-1}$, $x \neq 0$.

find $f'(x)$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^{-1} - (x)^{-1}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} \frac{\frac{-h}{x(x+h)}}{h}$$

$$= \lim_{h\to 0} -\frac{1}{x(x+h)} = -\frac{1}{x^2} = -1 \cdot x^{-2} = -1 \cdot x^{-1-1}$$

③ Sum, Difference & Constant multiple rules:

- $\left(f(x) \pm g(x)\right)' = f'(x) + g'(x)$
- $\left(c f(x)\right)' = c\, f'(x)$

Eg. $f(x) = 3x^5 + 10x^2 + 7$

Then $f'(x) = \left(3x^5 + 10x^2 + 7\right)'$

$= (3x^5)' + (10x^2)' + (7)'$

$= 3(x^5)' + 10(x^2)' + 0$, [Constant Mult. Rule & $(c)' = 0$]

$= 3\left(5x^{5-1}\right) + 10\left(2x^{2-1}\right)$ [power rule]

$= 15x^4 + 20x^1 = 15x^4 + 20x$

Now, $f''(x) = 15 \cdot 4x^{4-1} + 20 \cdot (1)$

$= 60x^3 + 20$

$f'''(x) = 60 \cdot 3x^{3-1} + 0$

$= 180x^2$

& so on.

Ex. Find the equation of the tangent of above $f(x)$

at $x = 0$ & $x = -1$

- We have $f'(x) = 15x^4 + 20x$

For tangent at $x=0$

So, $f(0) = 3(0)^5 + 10(0)^2 + 7$
$= 7$

$\Big\}$ tangent passes through $(0,7)$

$f'(0) = 15(0)^4 + 20(0)$
$= 0$

$\Big\}$ slope of tangent at $x=0$

So by point-slope form we get the tangent passing through $(0,7)$ with slope $= 0$ as

$$y-7 = 0(x-0)$$
$$\Rightarrow y-7=0 \Rightarrow \boxed{y=7}.$$

For tangent at $x=-1$

So, $f(-1) = 3(-1)^5 + 10(-1)^2 + 7$
$= -3+10+7$
$= 14$

$\Big\}$ tangent passes through $(-1,14)$

$f'(0) = 15(-1)^4 + 20(-1)$
$= 15-20$
$= -5$

$\Big\}$ slope of tangent at $x=-1$

So by point-slope form we get the tangent passing through $(-1,14)$ with slope $= -5$ as

$$y-14 = -5(x-(-1))$$
$$\Rightarrow y-14 = -5(x+1) = -5x-5$$
$$\Rightarrow y = -5x+14-5 = -5x+9$$
$$\Rightarrow \boxed{y=-5x+9}$$